

Pair creation of particles and black holes in external fields

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It is well known that massive black holes may form through the gravitational collapse of a massive astrophysical body. Less known is the fact that a black hole can be produced by the quantum process of pair creation in external fields. These black holes may have a mass much lower than their astrophysical counterparts. This mass can be of the order of Planck mass so that quantum effects may be important. This pair creation process can be investigated semiclassically using non-perturbative instanton methods, thus it may be used as a theoretical laboratory to obtain clues for a quantum gravity theory. In this work, we review briefly the history of pair creation of particles and black holes in external fields. In order to present some features of the euclidean instanton method which is used to calculate pair creation rates, we study a simple model of a scalar field and propose an effective one-loop action for a two-dimensional soliton pair creation problem. This action is built from the soliton field itself and the soliton charge is no longer treated as a topological charge but as a Noether charge. The results are also valid straightforwardly to the problem of pair creation rate of domain walls in dimensions $D \geq 3$.

1. Black hole pair creation

Nowadays we have good observational evidence for black holes with a mass range between one solar mass and 10^{10} solar masses. These massive black holes have been produced through the gravitational collapse of massive astrophysical bodies. One may be tempted to speculate on the possible existence of black holes of much lower mass (of the order of Planck mass) for which quantum effects can be important. However, such black holes could not form from the collapse of normal baryonic matter because degeneracy pressure will support white dwarfs or neutron stars below the Chandrasekhar limiting mass. Nevertheless, Planck size black holes may form through the tunneling quantum process of pair creation in external field.

This kind of process was first proposed for electron-positron pair creation in the vacuum only due to the presence of an external electric field. Because of the vacuum quantum fluctuations, virtual electron-positron pairs are constantly being produced and annihilated. These pairs can become real if they are pulled apart by an external electric field. The energy for the materialization and acceleration of the pair comes from a decrease of the external electric field

energy. In the same way, a black hole pair can be created in the presence of an external field whenever the energy pumped into the system is enough in order to make the pair of virtual black holes real. The energy for black hole pair creation can be provided by a heat bath of gravitons^{1,2}, by a background electric field³⁻⁶, by a background magnetic field^{7,8}, by a cosmic string⁹⁻¹¹, by a domain wall¹² or by a rapid cosmological expansion of the universe during the inflation era¹³⁻¹⁷.

Let us focus our attention on the process of black hole pair creation during inflation¹³⁻¹⁷. The inflationary era is not a good era to form black holes via gravitational collapse since matter is expanding away fast, rather than collapsing. However, this is a good era to create black holes through the quantum process of pair creation. The presence of large quantum fluctuations during inflation lead to strong gravitational perturbations and thus stimulates spontaneous black hole formation. Then, after the pair creation process, one has already a force present which pulls the pair apart. Black holes will be separated by the rapid cosmological expansion due to the effective cosmological constant, Λ_{eff} . So, the cosmological expansion during the inflationary era prevents the black hole production via gravitational collapse, but provides the background needed for their quantum pair creation.

Using the instanton method, the pair creation rate for this process can be calculated. Pairs of black holes with a typical radius $r_{\text{BH}} = 1/\Lambda_{\text{eff}}$ are produced with a rate (in Planck units) given by $\Gamma \propto \exp[-\pi/\Lambda_{\text{eff}}]$, so pair creation is suppressed. When $\Lambda_{\text{eff}} \approx 1$ (early in inflation), the suppression is weak and one can get a large number of black holes with a radius of order of Planck size. For smaller values of Λ_{eff} (later in inflation), black holes are created with larger radius but their creation becomes exponentially suppressed.

After being pair created, as the inflaton field rolls down, Λ_{eff} decreases, and so the black hole grows slowly ($r_{\text{BH}} = 1/\Lambda_{\text{eff}}$). However, the black hole also loses mass due to Hawking radiation and evaporates, so neutral black holes are highly suppressed after being pair created. The situation is different in the case of magnetically charged black holes which cannot evaporate, because either there are no magnetically charged particles they could radiate or they are very massive. In spite of this, the pair creation of magnetic black holes is so small and they are so diluted by the inflationary expansion that the probability to find one in our observable universe is extremely small.

We now briefly mention the other black hole creation processes. In the case of a heat bath of gravitons^{1,2} the creation process is not necessarily a pair creation. A single pair black hole can pop out from the thermal bath. In fact, due to statistical fluctuations, small black holes (with a temperature inversely proportional to the mass) can be produced in a thermal bath of gravitons. If

the black hole's temperature is higher than the temperature of the background thermal bath then the black holes will evaporate by Hawking radiation. However, if the black hole's temperature is smaller than the temperature of the background thermal bath, the black hole will increase its mass by accretion of matter.

For electrically ^{3–6} and magnetically ^{7,8} charged black holes, the electromagnetic force separates the recently created pair.

Black holes can also pair create in the background of a cosmic string ^{9–11} or domain wall ¹². In these cases the force that keeps the black holes apart comes from the string and domain wall tensions.

In what follows we present a brief description of other studies on black pair production. The process of black hole pair creation has been studied in relation to black hole entropy ^{18–21}, in de Sitter and AdS spacetimes ^{21–27}, in instanton manifolds ^{28,29}, in wormhole background ³⁰, in relation to Unruh effect ³¹, within the no-boundary proposal ³², in an inflating brane-world ³³, and in primordial black hole setting ^{34–36}.

2. Particle pair creation in external fields reviewed

In order to better understand the black hole pair creation process we study now the particle pair creation process in external fields. In this section, we present a brief historical review of this kind of process. Then, on the next section, we study a specific model of soliton pair creation in a 1+1 dimensional scalar field theory.

Klein ³⁷ has proposed the process of electron-positron pair creation in the vacuum due to the presence of an external electric field. This production process has been introduced in order to solve Klein's paradox, which is related to the fact that the reflected plus the transmitted flux are greater than the incident flux when one considers the solution of Dirac's equation for an electron entering into a region subjected to an external electric field. Sauter ³⁸ has shown that in order to materialize this pair, one has to have that the potential energy must satisfy $\Delta V = eE\Delta l \geq 2mc^2$ during approximately one Compton length, $\Delta l \sim \hbar/mc$, so that the critical value for the electric field that one needs for the creation process is $E_{\text{cr}} \sim 2.6 \times 10^{26} \text{Vcm}^{-1}$. Heisenberg and Euler ³⁹ have proposed, in the framework of electron-hole theory, an one-loop effective lagrangian that accounts for the vacuum fluctuations effects and with it have calculated the electron-positron pair creation rate. Later, Schwinger ⁴⁰ has obtained the same result using a field theory approach by making use of his proper time method.

Langer ⁴¹, in 1967, in his work about decay of metastable thermodynamical

states, has introduced the powerful euclidean instanton method. As noticed by Stone ⁴⁴, one can regard the external field as a false vacuum since its energy can be lowered by creating a pair of sufficiently separated particles. The semiclassical instanton method and Stone's interpretation have been applied to several different studies namely: Coleman and Callan ^{42,43} have computed the bubble production rate that accompanies the cosmological phase transitions in a (3+1)D scalar field theory; Stone ⁴⁴, Kiselev and Selivanov ^{45,46} and Voloshin ⁴⁷ have calculated the soliton pair creation rate that accompanies the decay of a metastable vacuum on a (1+1)D scalar field theory; Affleck and Manton ⁴⁸ have studied monopole pair production in a weak external magnetic field; Affleck, Alvarez and Manton ⁴⁹ have worked on e^+e^- boson pair production in a weak external electric field and finally the studies on black hole pair production in external fields ¹⁻³⁶.

For all these processes the instanton method can be used to compute the pair creation rate, which is generally given by $\Gamma = A \exp[-(S_{\text{pair}}^{\text{cl}} - S_{\text{backg}}^{\text{cl}})]$. Here, $S_{\text{pair}}^{\text{cl}}$ is the classical action of the instanton mediating the pair creation, $S_{\text{backg}}^{\text{cl}}$ is the classical action of the background field alone and pre-factor A is the one-loop contribution which includes the quantum corrections.

More recently, Miller and his collaborators have presented quite interesting experimental evidence for quantum pair creation of charged solitons in a condensed matter system ^{50,51}.

3. The Instanton method. Effective one-loop action for pair creation of domain walls

Stone ⁴⁴ has studied the problem of a scalar field theory in (1+1)D with a metastable vacuum, i.e., with a scalar potential U that has a false vacuum, ϕ_+ , and a true vacuum, ϕ_- , separated by an energy density difference, ϵ . Stone has noticed that the decay process can be interpreted as the false vacuum decaying into the true vacuum plus a creation of a soliton-antisoliton pair: $\phi_+ \rightarrow \phi_- + s + \bar{s}$. The dynamics is governed by the action,

$$S[\phi(x, t)] = \int d^2x \left[\frac{1}{2} \partial_\mu \phi \partial^\mu \phi + U(\phi) \right]. \quad (1)$$

We have proposed ⁵² an euclidean effective one-loop action for Stone's problem which is built from the soliton field itself given by:

$$S_{\text{Euc}}^{\text{eff}}(\psi) = \int d^2x \left[\left| \left(\partial_\mu - \frac{1}{2} \epsilon \varepsilon_{\mu\nu} x_\nu \right) \psi \right|^2 + m^2 |\psi|^2 \right]. \quad (2)$$

The action consists of the usual mass term and a kinetic term in which the simple derivative of the soliton field is replaced by a kind of covariant derivative. In this effective action the soliton charge is treated no longer as a topological charge but as a Noether charge. This procedure of working with an effective action for the soliton field itself is not new. Coleman⁵³ has shown the equivalence between the Sine-Gordon model and the Thirring model. In this picture, the Sine-Gordon soliton is represented by the local Fermi field of the Thirring model and there is an interchange of Noether and topological charges. Montonen and Olive⁵⁴ have proposed an equivalent dual field theory for the Prasad-Sommerfield monopole soliton⁵⁵ in which the fundamental monopole solitons fields play the role of heavy gauge particles, with the Prasad-Sommerfield topological magnetic charge being now a Noether charge. Manton⁵⁶ has proposed an effective action built from the soliton field itself which reproduces the solitons' physical properties of (1+1)D nonlinear scalar field theories that have symmetric potentials with degenerate minima. In the present problem one deals with (1+1)D scalar field theories which have a potential with non-degenerate minima, so our effective action is new since Manton was not dealing with the soliton pair production process.

Now, we can use the semiclassical instanton method to calculate the soliton-antisoliton pair production rate per unit time. Following Stone's interpretation this is equal to the decay rate per unit time of the false vacuum ($\hbar = c = 1$):

$$\Gamma = -2 \operatorname{Im} E_0 . \quad (3)$$

One can see that it is so by considering the wavefunction associated with the metastable vacuum energy and analysing its probability evolution along the time. The vacuum energy, E_0 , is given by the euclidean path integral:

$$e^{-E_0 T} = \lim_{T \rightarrow \infty} \int [\mathcal{D}\psi][\mathcal{D}\psi^*] e^{-S_{\text{Euc}}^{\text{eff}}(\psi, \psi^*)} , \quad (4)$$

As it will be verified, E_0 will receive a small imaginary contribution from the negative-mode associated to the quantum fluctuations about the instanton (which stationarizes the action) and this fact is responsible for the decay. After some calculations that make use of the "Schwinger proper time integral" the creation rate can be written as:

$$\Gamma = \lim_{T \rightarrow \infty} \frac{1}{T} \frac{2}{m} \sqrt{\frac{2\pi}{T_0}} \operatorname{Im} \int [dx] e^{-S_{\text{Euc}}[x_\mu(\tau)]} , \quad (5)$$

where $S_{\text{Euc}} = m \sqrt{\int_0^1 d\tau \dot{x}_\mu \dot{x}_\mu} + \frac{1}{2} \epsilon \oint \varepsilon_{\mu\nu} x_\nu dx_\mu$ is now an effective action for a particle moving subjected to an external field in a (2+1) dimensional spacetime, with the Schwinger proper time τ playing the role of time coordinate.

The classical solution of the equation of motion is called the instanton, $x_\mu^{\text{cl}}(\tau) = R(\cos 2\pi\tau, \sin 2\pi\tau)$, and represents a particle describing a loop of radius R along the proper time. The loop has a thin wall separating the true vacuum within from the false vacuum outside. The euclidean action of the instanton is given by $S_0 = S_{\text{Euc}}[x_\mu^{\text{cl}}(\tau)] = m2\pi R - \epsilon\pi R^2$. The first term is the rest energy of the particle times the orbital length and the second term represents the interaction of the particle with the external scalar field.

Now, to include the quantum effects, small fluctuations about the instanton are considered, i.e., the expansion $x_\mu(\tau) = x_\mu^{\text{cl}}(\tau) + \eta_\mu(\tau)$. The euclidean action is expanded to second order so that the path integral (5) can be approximated by:

$$\Gamma \simeq \lim_{T \rightarrow \infty} \frac{1}{T} \frac{2}{m} \sqrt{\frac{2\pi}{T_0}} e^{-S_0} \text{Im} \int [d\eta(\tau)] \exp \left[-\frac{1}{2} \int d\tau d\tau' \eta_\mu(\tau) M_{\mu\nu} \eta_\nu(\tau') \right] \quad (6)$$

The path integral in equation (6) is called the one-loop factor and is given by $\mathcal{N}(\text{Det} M)^{-\frac{1}{2}} = \mathcal{N} \prod (\lambda_n)^{-\frac{1}{2}}$, where λ_n are the eigenvalues of $M_{\mu\nu}$, the second order variation operator of the action. Besides an infinite number of positive eigenvalues, one has two zero eigenvalues associated with the translation of the loop along the x_1 and x_2 directions plus a zero eigenvalue associated with the translation along the proper time, τ . There is also a single negative mode associated to the change of the loop radius. Note that it is this single negative eigenvalue, when one takes its square root, that is responsible for the imaginary contribution to the creation rate.

To overcome the problem of having a product of an infinite number of eigenvalues one has to compare our system with the background system without the pair created. In the product, one omits the zero eigenvalues, but one has to introduce the normalization factor $\frac{||dx_\mu^{\text{cl}}/d\tau||}{||\eta_\mu^0||} \sqrt{\frac{1}{2\pi}}$ which is associated with the proper time eigenvalue. In addition, associated with the negative eigenvalue one has to introduce a factor of 1/2 which accounts for the loops that do expand. The other 1/2 contracts (representing the annihilation of recently created pairs) and so does not contribute to the creation rate. One also has to introduce the spacetime volume factor $\int dx_2 \int dx_1 = TL$, which represents the spacetime region where the instanton might be localized.

Finally, the soliton-antisoliton pair production rate per unit time and length is given by ⁵²:

$$\Gamma/L = \frac{\epsilon}{2\pi} e^{-\frac{\pi m^2}{\epsilon}}. \quad (7)$$

With our effective action (2) we have recovered Stone's exponential factor

$e^{-\frac{\pi m^2}{\epsilon}}$ ⁴⁴ and the pre-exponential factor $A = \epsilon/2\pi$ of Kiselev, Selivanov and Voloshin ^{45,46,47}.

One can make an analytical continuation of the euclidean time back to the Minkowskian time and obtain the solution in 2D Minkowski spacetime which tells us that at $t=0$ the system makes a quantum jump and as a consequence of it a soliton-antisoliton pair materializes at $x = \pm R$. After the materialization, the soliton and antisoliton are accelerated, driving away from each other. The energy needed for this process comes from the energy released when the false vacuum is converted into true vacuum in the region between the soliton pair.

It is well known that a one-particle system in 2D can be transformed straightforwardly to a thin line in 3D and a thin wall in 4D, where now the mass m of the soliton should be interpreted as a line density and surface density, respectively. (In fact a particle in (1+1)D, as well as an infinite line in (2+1)D, can be considered as walls as seen from within the intrinsic space dimension, justifying the use of the name wall for any dimension). Our calculations apply directly to the domain wall pair creation problem in any dimension.

4. Conclusions

In this work we have reviewed the possibility of producing Planck size black hole pairs through the quantum tunneling process of pair creation. We have seen the principal features of the semiclassical instanton method which is used to calculate particle pair creation rates in external fields. In particular, we have seen that the creation rate is given by the imaginary part of a path integral. The instanton is the classical solution that stationarizes the euclidean action. Quantum corrections are included in the one-loop factor when one considers the quantum fluctuations around the instanton. An usual characteristic of the one-loop factor is the presence of: (i) an infinite product of eigenvalues; (ii) zero eigenvalues; (iii) a negative eigenvalue. It is this last one which is responsible for the creation rate.

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